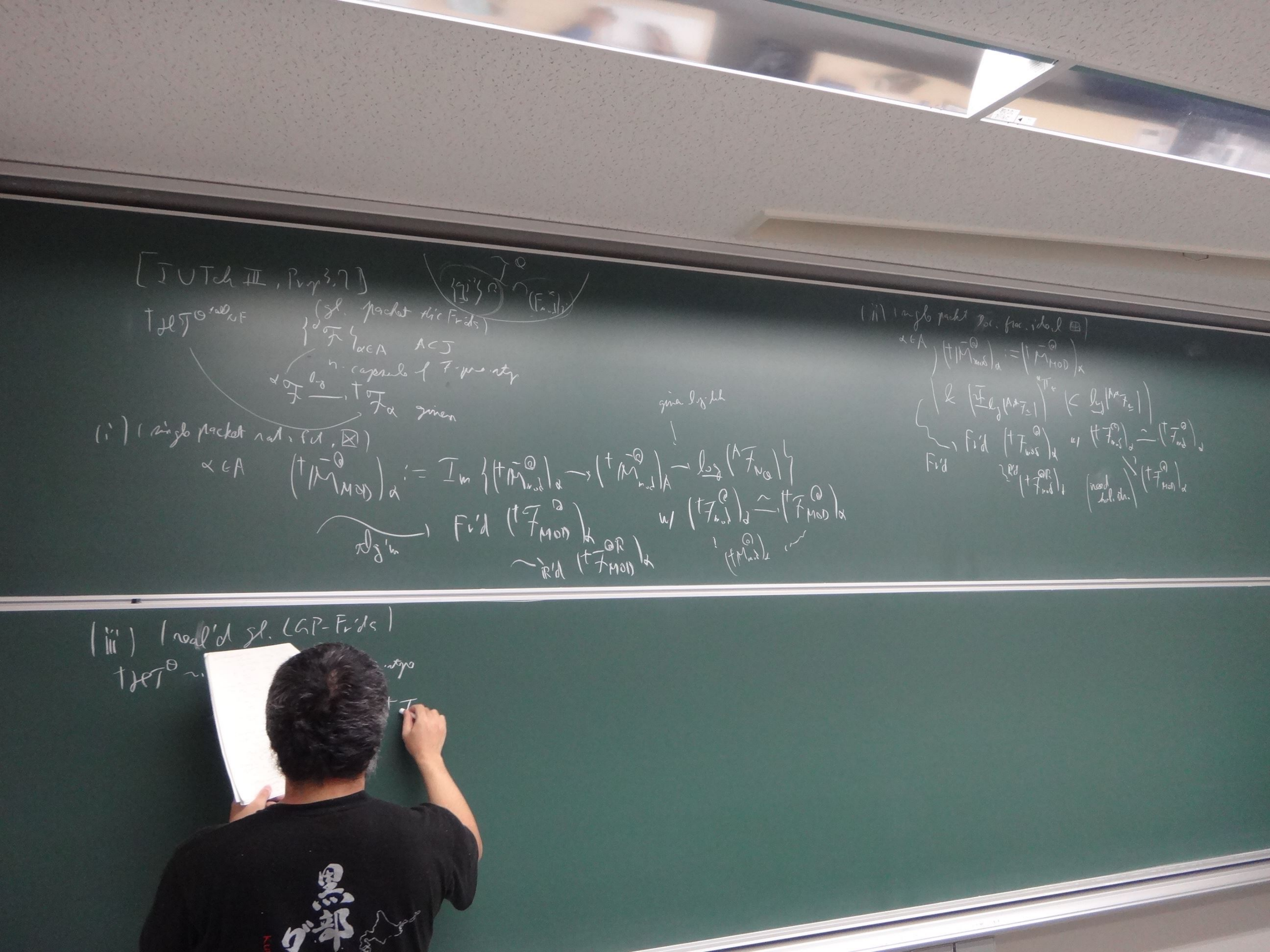
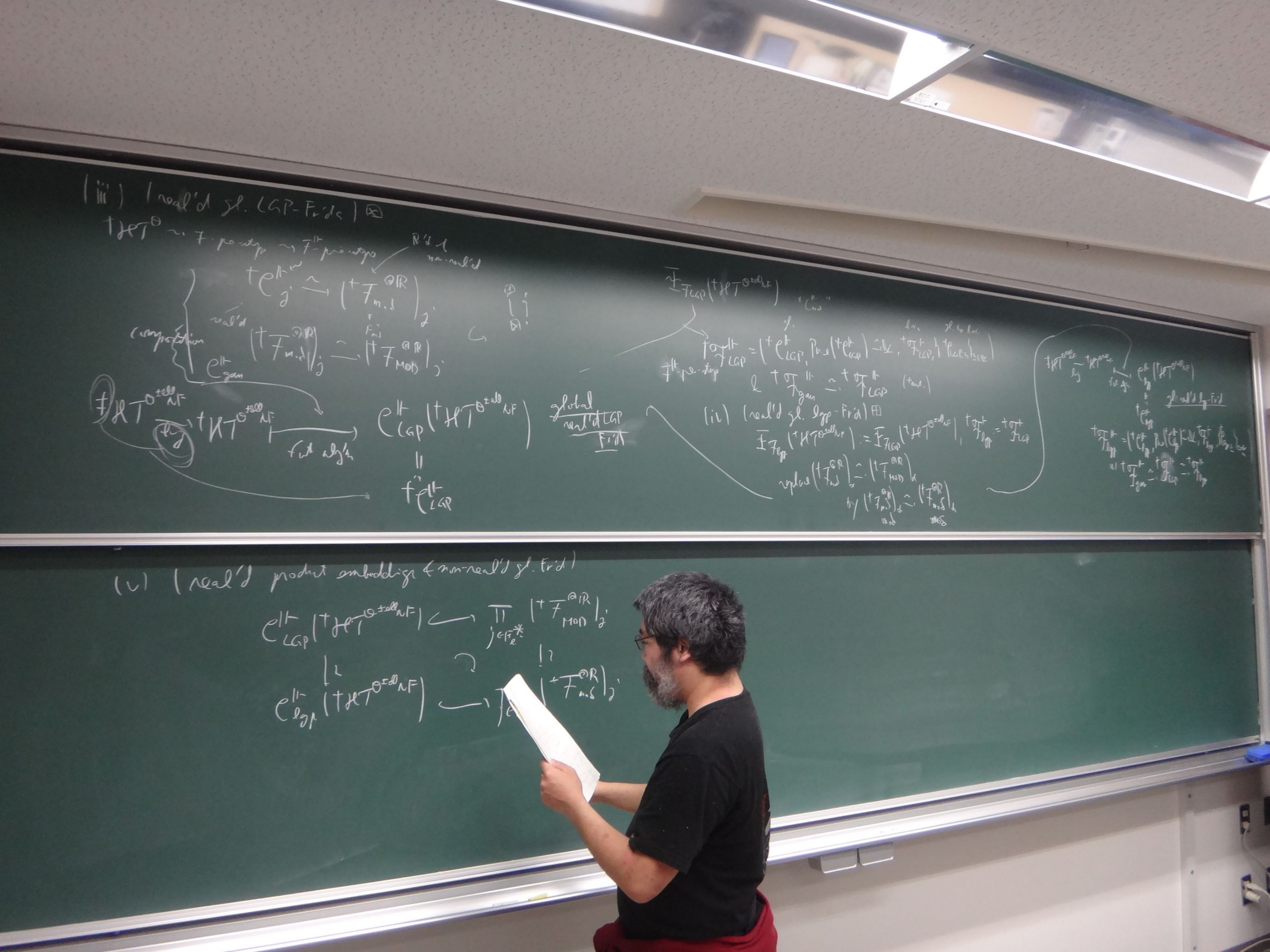

[IUTch III, Prop 3.7]
(i) (single packet nat. fil. ⊠)
(ii) (single packet loc. frac. ideal ⊞)
(iii) (real'd gl. LGP-Fr'ds)


(iii) (real'd gl. LGP-Fr'ds)
global real'd LGP Fr'd
(iv) (real'd gl. lgp-Fr'd)
replace
by
(v) (real'd product embeddings & non-real'd gl. Fr'd)
composition
real'd

(v) (real'd product embeddings & non-real'd gl. Frob'd)

$$\begin{array}{ccc}
\mathcal{C}^{\Vdash}_{\mathrm{LGP}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}) & \hookrightarrow & \prod_{j\in\mathbb{F}_l^*} ({}^{\dagger}\mathcal{F}^{\circledast\mathbb{R}}_{\mathrm{MOD}})_j \\
\wr & \circlearrowright & \wr \\
\mathcal{C}^{\Vdash}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}) & \hookrightarrow & \prod_{j\in\mathbb{F}_l^*} ({}^{\dagger}\mathcal{F}^{\circledast\mathbb{R}}_{\mathrm{mod}})_j
\end{array}$$

" $a \in K_{\sim}^{\times}$

q^{j^2} $a\mathcal{O}_{K_{\sim}}$;

frac'l. ideal gen. by elts in $\mathcal{F}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}})^{{}^{\dagger}\Pi_{\sim}}$

$\wr$

obtain obj in $\mathcal{C}^{\Vdash}_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}})$

$\mu_{\mathbb{N}}$

${}^{\dagger}\mathcal{L}^{\circledcirc}$

${}^{\dagger}\mathcal{D}^{\circledast\pm}$

[IUTch III, Def 3.8]

(i) $\prod_{j\in\mathbb{F}_l^*} ({}^{\dagger}\mathcal{F}^{\circledast}_{\mathrm{mod}})_j$

$\cup$

gen. ob

$\circledast \leftarrow \mathcal{F}^{\perp}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}})$

$\circledast \in \mathrm{Ob}({}^{\dagger}\mathcal{C}^{\Vdash}_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}})$

η to for "$\underline{\underline{q}}^{j^2}$"

$\underline{\text{(b)-pilot obj}}$

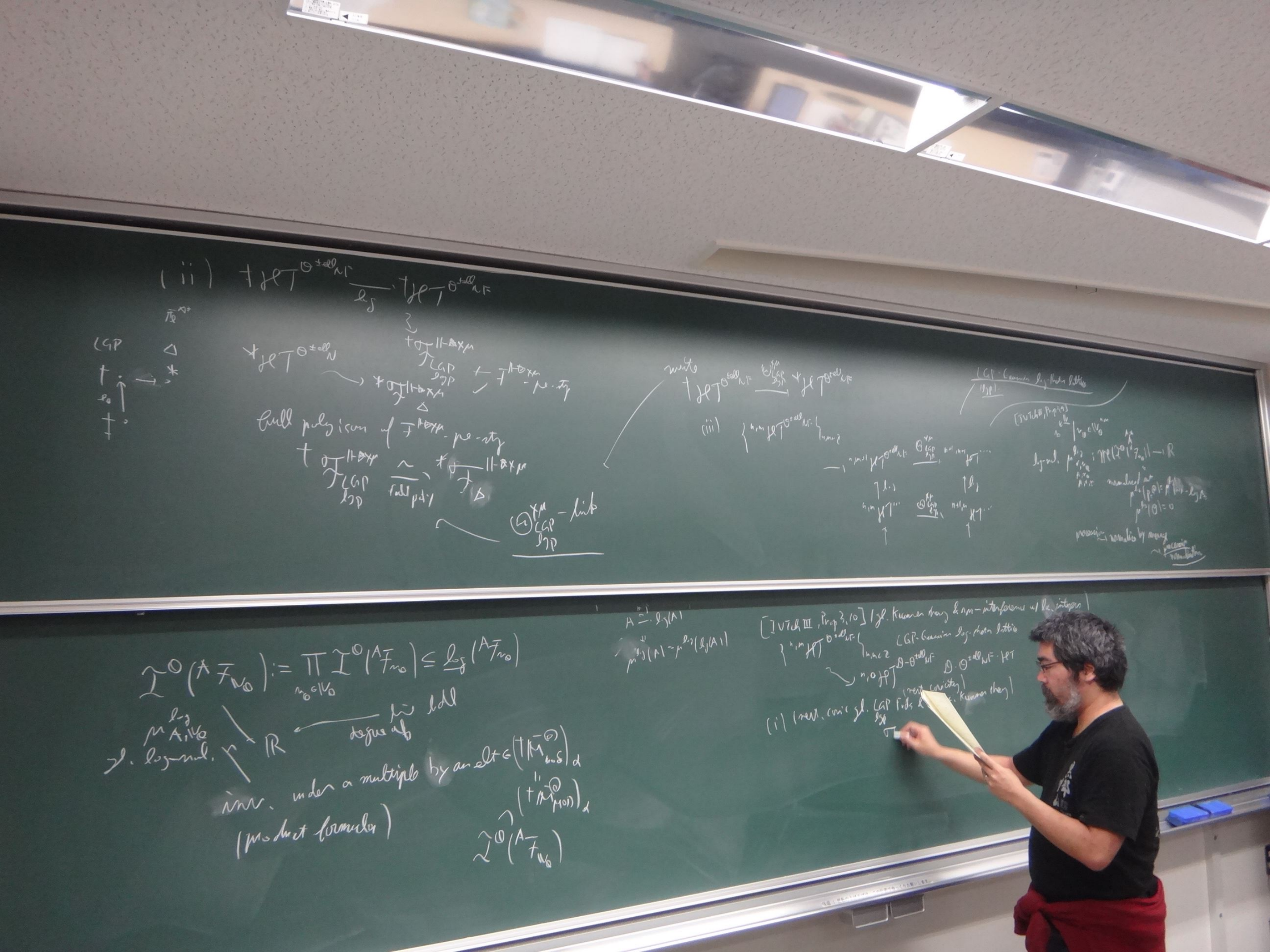
${}^{\dagger}\mathcal{C}^{\Vdash}_{\Delta}$ $\quad \Delta = \{0, \langle \mathbb{F}_l^* \rangle\}$

splitting monoid $({}^{\dagger}\mathcal{F}^{\vdash}_{\Delta})_{\sim}$

$\rightsquigarrow$ gen. up to tor

$\rightsquigarrow \circledast \in \mathrm{Ob}({}^{\dagger}\mathcal{C}^{\Vdash}_{\Delta})$ "$\underline{\underline{q}}$"

$\underline{\text{q-pilot obj}}$


(ii)
LGP
full poly-isom of
write
(iii)
LGP-Gaussian log-theta lattice
normalized
[IUTchIII, Prop 3.10] / gl. Kummer theory & non-interference w/ the integers
LGP-Gaussian log-theta lattice
(i) (mult. conic gl. LGP Frobs
degree of
gl. log-vol.
inv. under a multiple by an elt
(product formula)

$\mathcal{I}^{\circledcirc}({}^{\wedge}\mathcal{F}_{\mathbb{V}_{\mathbb{Q}}}) := \prod_{v_{\mathbb{Q}} \in \mathbb{V}_{\mathbb{Q}}} \mathcal{I}^{\circledcirc}({}^{\wedge}\mathcal{F}_{v_{\mathbb{Q}}}) \subseteq \log({}^{\wedge}\mathcal{F}_{v_{\mathbb{Q}}})$

$\mu^{\log}_{A, \mathbb{V}_{\mathbb{Q}}}$

gl. log-vol. $\curvearrowright \mathbb{R} \longleftarrow$ degree of ... ar. ldl

inv. under a multiple by an elt $\in ({}^{\dagger}\mathcal{M}^{\circledcirc}_{\mathrm{mod}})_{\alpha}$

(product formula)

$({}^{\dagger}\mathcal{M}^{\circledcirc}_{\mathrm{MOD}})_{\alpha} \curvearrowright \mathcal{I}^{\circledcirc}({}^{\wedge}\mathcal{F}_{\mathbb{V}_{\mathbb{Q}}})$

$A \overset{\sim}{\to} \log(A)$

$\mu^{\log}(A) = \mu^{\log}(\log(A))$

[IUTchIII, Prop 3.10] (gl. Kummer theory & non-interference with loc. integers)

$\{{}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF}\}_{n,m \in \mathbb{Z}}$ (LGP: Gaussian log-theta lattice)

$\leadsto {}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF}$, $D\text{-}\Theta^{\pm ell}NF\text{-}\mathcal{HT}$

(i) (vert. coric gl. LGP-fields & assoc. Kummer theory) (vert. coricity)

$\mathcal{F}({}^{n,\circ}\mathfrak{S})_{\alpha}$

et-like: ${}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF} \longmapsto$ | field $\mathcal{M}^{\circledcirc}_{\mathrm{mod}}$, MOD $({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF})_{\alpha}$

Frob-like: Frd $\overline{\mathcal{F}}^{\circledcirc}_{\mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF})_{\alpha} \overset{\sim}{\to} \overline{\mathcal{F}}^{\circledcirc}_{\mathrm{mod}}(\ldots)_{\alpha} \overset{\sim}{\to} \overline{\mathcal{F}}^{\circledcirc}_{\mathrm{MOD}}(\ldots)_{\alpha}$

$\alpha \in A[\,]$

vert. coric: $\mathcal{F}^{\Vdash}$-prime-strip $\mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{gau}} \overset{\sim}{\to} \mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{LGP}} \overset{\sim}{\to} \mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{lgp}}$

(ii) ${}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow{\log} {}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF}$

${}^{\dagger}\mathcal{F}^{\Vdash\blacktriangle\times\mu}_{\mathrm{LGP}}$, ${}^{\dagger}\mathcal{F}^{\Vdash\blacktriangle\times\mu}_{\mathrm{lgp}} \longleftarrow \mathcal{F}^{\Vdash\blacktriangle\times\mu}$-prime-strip

${}^{*}\mathcal{HT}^{\Theta^{\pm ell}NF} \leadsto {}^{*}\mathcal{F}^{\Vdash\blacktriangle\times\mu}_{\Delta}$

LGP: $\mathbb{F}^{\times\mu}_{l}$, Δ

${}^{\dagger}\cdot \longrightarrow {}^{*}$

$\log$: ${}^{\dagger}\cdot \uparrow {}^{\ddagger}\cdot$

full poly-isom of $\mathcal{F}^{\Vdash\blacktriangle\times\mu}$-prime-strip

${}^{\dagger}\mathcal{F}^{\Vdash\blacktriangle\times\mu}_{\mathrm{LGP}, \mathrm{lgp}} \xrightarrow[\text{full poly}]{\sim} {}^{*}\mathcal{F}^{\Vdash\blacktriangle\times\mu}_{\Delta}$

$\Theta^{\times\mu}_{\mathrm{LGP}, \mathrm{lgp}}$-link

$\Theta^{\blacktriangle}$, ${}^{*}\Theta^{\times\mu}$

write ${}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow{\Theta^{\times\mu}_{\mathrm{LGP}, \mathrm{lgp}}} {}^{\ddagger}\mathcal{HT}^{\Theta^{\pm ell}NF}$

(iii) $\{{}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF}\}_{n,m \in \mathbb{Z}}$

$\longrightarrow {}^{n,m+1}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow{\Theta^{\times\mu}_{\mathrm{LGP}, \mathrm{lgp}}} {}^{n+1,m+1}\mathcal{HT}\cdots$

$\uparrow \log$ $\quad \uparrow \log$

${}^{n,m}\mathcal{HT}\cdots \xrightarrow{\Theta^{\times\mu}_{\mathrm{LGP}, \mathrm{lgp}}} {}^{n+1,m}\mathcal{HT}\cdots$

$\langle \mathbb{F}_{l} \rangle$, $\{0, \langle \mathbb{F}^{*}_{l} \rangle\}$

LGP: Gaussian log-theta lattice

lgp:

[IUTchIII, Prop 3.9] $\mathbb{V}_{\mathbb{Q}}$, $\mathbb{V}^{\mathrm{non}}_{\mathbb{Q}}$, ${}^{\wedge}A$

log-vol. $\mu^{\log}_{A, \mathbb{V}_{\mathbb{Q}}}$; $\mathrm{pr}(\mathcal{I}^{\circledcirc}({}^{\wedge}\mathcal{F}_{v_{\mathbb{Q}}}))$ ...

normalized ...

$\mu^{\log}(\mathcal{I}^{\circledcirc}) = \ldots$

procession-normalization by averaging ...

$$\mathcal{I}^{\circledcirc}({}^{A}\mathcal{F}_{V_0}) := \prod_{v_0 \in V_0} \mathcal{I}^{\circledcirc}({}^{A}\mathcal{F}_{v_0}) \subseteq \log({}^{A}\mathcal{F}_{v_0})$$

$\mu^{\log}_{A, V_0}$

gl. log-vol. $\mathbb{R}$ ← degree of ... ldl

inv. under a multipl. by an elt $\in (\dagger \mathbb{M}^{\circledcirc}_{\mathrm{mod}})_{\alpha}$

(product formula)

$\wr\wr$

$(\dagger \mathbb{M}^{\circledcirc}_{\mathrm{MOD}})_{\alpha}$

$\curvearrowright \tilde{\mathcal{I}}^{\circledcirc}({}^{A}\mathcal{F}_{V_0})$

$A \overset{n \cdot j}{\cong} \log(A)$

$\mu^{\log}(A) = \mu^{\log}(\log(A))$

[IUTch III, Prop. 3.10] (gl. Kummer theory & non-interference w/ loc. integers)

$\{ {}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}\mathrm{NF}} \}_{n,m \in \mathbb{Z}}$ (LGP-Gaussian log-theta lattice)

$\rightsquigarrow {}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}}$ $\quad \mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}\text{-}\mathcal{HT}$

(i) (vert. coric gl. LGP/lgp Frobs & assoc. Kummer theory) (vert. coricity)

$\mathcal{F}({}^{n,\circ}\mathfrak{S})_{\alpha}$

et-like

${}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}} \longmapsto$ (F..alg's, $\alpha \in A[\ ]$, vert. coric)

- field monoid: $\mathbb{M}^{(\sim)\circledcirc}_{\mathrm{mod}/\mathrm{MOD}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}})_{\alpha}$
- Fr'd: $\mathcal{F}^{\circledcirc}_{\mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}})_{\alpha} \cong \mathcal{F}^{\circledcirc}_{\mathrm{mod}}(\ldots)_{\alpha} \cong \mathcal{F}^{\circledcirc}_{\mathrm{MOD}}(\ldots)_{\alpha}$
- $\mathcal{F}^{\Vdash}$-pr-strip: $\mathcal{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{gau}} \cong \mathcal{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{LGP}} \cong \mathcal{F}^{\Vdash}({}^{n,\circ}\ldots)_{\mathrm{lgp}}$

Kummer → Frob

NF monoid: $({}^{n,m}\mathbb{M}^{(\sim)\circledcirc}_{\mathrm{mod}/\mathrm{MOD}/\mathrm{mod}})_{\alpha} \overset{\mathrm{Kmm}}{\cong} \mathbb{M}^{(\sim)\circledcirc}_{\mathrm{mod}/\mathrm{MOD}/\mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}})_{\alpha}$

Fr'd: $({}^{n,m}\mathcal{F}^{\circledcirc}_{\mathrm{mod}/\mathrm{MOD}/\mathrm{mod}})_{\alpha} \overset{\mathrm{Kmm}}{\cong} \mathcal{F}^{\circledcirc}_{\mathrm{mod}/\mathrm{MOD}/\mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}})_{\alpha}$

$\mathcal{F}^{\Vdash}$-pr-strip: ${}^{n,m}\mathcal{F}^{\Vdash}_{\mathrm{gau}/\mathrm{LGP}/\mathrm{lgp}} \overset{\mathrm{Kmm}}{\cong} \mathcal{F}^{\Vdash}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}\mathrm{NF}})_{\mathrm{gau}/\mathrm{LGP}/\mathrm{lgp}}$

$\mathcal{I}^{\circledcirc}$

$\{q^{j^2}\} \subseteq \mathcal{I}(F^{\times}_{\mathrm{mod}})$

$F^{\times}_{\mathrm{mod}} \cap \prod_{v} \mathcal{O}^{\circledcirc}_{v} = \mu$

(ii) (non-interference w/ local integers)

$$({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}})_{\alpha} \cap \prod_{\underline{v} \in \underline{\mathbb{V}}} \Phi_{\log}({}^{A,\alpha;n,m}\mathcal{F}_{\underline{v}}) = ({}^{n,m}\mathbb{M}^{\circledcirc\mu}_{\mathrm{MOD}})_{\alpha}$$

$$\left(\subseteq \prod_{\underline{v} \in \underline{\mathbb{V}}} \mathcal{I}^{\circledcirc}({}^{A,\alpha}\mathcal{F}_{\underline{v}})\right)$$

$({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}})_{j} \curvearrowright \tilde{\mathcal{I}}^{\circledcirc}(\mathfrak{S}^{\pm}_{j+1}\mathcal{F}({}^{n,\circ}\mathfrak{S})_{j}|_{V_0})$

via $\mathrm{Kmm} \circ (\log)^{m}_{m \geq 0}$

totality of these actions

Kummer $\}$

Frob.

NF monoid

$$\left({}^{n,m}\mathbb{M}^{(-)\circledcirc}_{\substack{\mathrm{mod} \\ \mathrm{MOD} \\ \mathrm{mod}}}\right)_\alpha \xrightarrow[\text{Kmm}]{\sim} \mathbb{M}^{(-)\circledcirc}_{\substack{\mathrm{mod} \\ \mathrm{MOD} \\ \mathrm{mod}}}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}}\right)_\alpha$$

ét

Fr'd

$$\left({}^{n,m}\mathcal{F}^{\circledcirc}_{\substack{\mathrm{mod} \\ \mathrm{MOD} \\ \mathrm{mod}}}\right)_\alpha \xrightarrow[\text{Kmm}]{\sim} \mathcal{F}^{\circledcirc}_{\substack{\mathrm{mod} \\ \mathrm{MOD} \\ \mathrm{mod}}}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}}\right)_\alpha$$

$\mathcal{F}^{\Vdash}$-pr-str.

$${}^{n,m}\mathfrak{F}^{\Vdash}_{\substack{\mathrm{gau} \\ \mathrm{LGP} \\ \mathrm{lgp}}} \xrightarrow[\text{Kmm}]{\sim} \mathfrak{F}^{\Vdash}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}}\right)_{\substack{\mathrm{gau} \\ \mathrm{LGP} \\ \mathrm{lgp}}}$$

(ii) (non-interference w/ local integers)

$$\mathcal{I}^{\mathbb{Q}}\left(\{q^{j^2}_{\underline{v}}\} \subseteq \mathcal{O}_{(F^{\times}_{\mathrm{mod}})_j}\right)$$

" $F^{\times}_{\mathrm{mod}} \cap \prod_{\underline{v}} \mathcal{O}_{\underline{v}} = \mu$ "

$$\left({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}}\right)_\alpha \cap \prod_{\underline{v} \in \underline{\mathbb{V}}} \Psi_{\log}\left({}^{\mathbb{A},\alpha;n,m}\mathcal{F}_{\underline{v}}\right) = \left({}^{n,m}\mathbb{M}^{\circledcirc\mu}_{\mathrm{MOD}}\right)_\alpha$$

$$\left(\subseteq \prod_{\underline{v} \in \underline{\mathbb{V}}} \mathcal{I}^{\mathbb{Q}}\left({}^{\mathbb{A},\alpha}\mathcal{F}_{\underline{v}}\right)\right)$$

$$\left({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}}\right)_j \curvearrowright \tilde{\mathcal{I}}^{\mathbb{Q}}\left(\mathbb{S}^{\pm}_{j+1}\mathcal{F}\left({}^{n,\circ}\mathfrak{D}_{\succ}\right)_{\underline{v}_0}\right)$$

via Kmm $\circ$ $(\log)^{m'}$, $m' \geq 0$

totality of these actions

mutually compat.

log-Kummer correspondence

($m \in \mathbb{Z}$)

j ship-ind.

(iii) (Fr'd th'c log-Kummer corresp.)

$$\left({}^{n,m}\mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}}\right)_\alpha \xrightarrow[\text{Kmm}]{\sim} \mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}}\right)_\alpha$$

mut. compat. in the above sense

(iii) (Frob'd th'c log-Kummer corresp.)

$$\left({}^{n,m}\mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{MOD}}\right)\Big|_{\alpha} \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{MOD}}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)\Big|_{\alpha}$$

$(m \in \mathbb{Z})$ mut. compat. in the above sense

$$(n \in \mathbb{Z}) \quad {}^{n,m}\mathcal{F}^{\Vdash\perp}_{\mathrm{LGP}} \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{F}^{\Vdash\perp}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)\Big|_{\mathrm{LGP}}$$

MOD, LGP

~~mod, lgp~~

[IUTch III, Th3.11] (multiradial algorithm via LGP-monoids/Frobs)

initial Θ-data $(\overline{F}/F, X_F, l, \underline{C}_K, \underline{\mathbb{V}}, \mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}}, \underline{\epsilon})$ fix

$\rightsquigarrow \{{}^{n,m}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\}_{n,m\in\mathbb{Z}}$ LGP-Gaussian log-theta lattice

$\hookrightarrow {}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}$ resh. assoc. $\mathcal{D}\Theta^{\pm\mathrm{ell}}\mathrm{NF}$-$\mathcal{HT}$ up to isom

$l^{\pm} = \frac{l+1}{2}$ $\quad \Delta^{\pm}$

(i') (multirad. rep'n)

$$\mathrm{Prc}\left({}^{n,\circ}\mathfrak{S}^{\pm}_{T}\right) : \{{}^{n,\circ}\mathfrak{S}^{\pm}_{0}\} \hookrightarrow \{{}^{n,\circ}\mathfrak{S}^{\pm}_{0}, {}^{n,\circ}\mathfrak{S}^{\pm}_{1}\} \hookrightarrow \cdots \hookrightarrow \{{}^{n,\circ}\mathfrak{S}^{\pm}_{0}, \ldots, {}^{n,\circ}\mathfrak{S}^{\pm}_{l^{*}}\}$$

consider the following data

(a) $\underline{\mathbb{V}} \ni \underline{v} \mid v_{\mathbb{Q}},\ j \in |\mathbb{F}_l|$ $\quad$ log. vol. data $\quad \mathcal{I}\left({}^{\mathbb{S}^{\pm}_{j+1};\,n,\circ}\mathcal{D}^{\vdash}_{v_{\mathbb{Q}}}\right) \subset \mathcal{I}^{\mathbb{Q}}\left({}^{\mathbb{S}^{\pm}_{j+1};\,n,\circ}\mathcal{D}^{\vdash}_{v_{\mathbb{Q}}}\right)$

mono-anal. data

w/ proc. normalized log-vol.

(A, 1) (AA, 1)

(b) $\underline{\mathbb{V}}^{\mathrm{bad}} \ni \underline{v}$

splitting monoid

$$\underline{\underline{\mathcal{F}}}^{\perp}_{\mathrm{LGP}}\left({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)\Big|_{\underline{v}}$$

$$\cap$$

$$\prod_{j\in\mathbb{F}^{*}_{l}} \mathcal{I}^{\mathbb{Q}}\left({}^{\mathbb{S}^{\pm}_{j+1};\,n,\circ}\mathcal{D}^{\vdash}_{\underline{v}}\right)$$

via

(h). $\underline{\mathbb{V}}^{\text{bad}} \ni \underline{v}$

splitting monoid

$$\underline{\underline{\mathcal{F}}}^{\vdash\perp}_{\mathrm{LGP}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}})|_{\underline{v}}$$

w/ action on

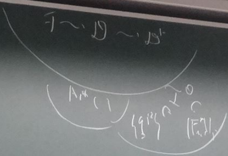
$$\bigcup$$

$$\prod_{j \in \mathbb{F}_l^{*}} \mathcal{I}^{\mathbb{Q}}({}^{S^{\pm}_{j+1};\, j;\, n,\circ}\mathcal{D}^{\vdash}_{\underline{v}})$$

via

$$\mathcal{I}^{\mathbb{Q}}(\cdots \mathcal{D}^{\vdash}_{\underline{v}}) \underset{\text{poly}}{\overset{\sim}{\to}} \mathcal{I}^{\mathbb{Q}}(\cdots \mathcal{F}^{\vdash\times\mu}({}^{n,\circ}\mathcal{S})_{\xi}|_{\underline{v}}) \simeq \mathcal{I}^{\mathbb{Q}}(\cdots \mathcal{F}({}^{n,\circ}\mathcal{S})_{\xi}|_{\underline{v}})$$

Isomet-orbit



(c). $j \in \mathbb{F}_l^{*}$

NF $\quad \overline{\mathbb{M}}^{\circledast}_{\mathrm{MOD}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}})_j$

$$= \overline{\mathbb{M}}_{\mathrm{mod}}(\ -\ )$$

$$\subseteq \mathcal{I}^{\mathbb{Q}}({}^{S^{\pm}_{j+1};\, n,\circ}\mathcal{D}^{\vdash}_{V_0}) := \prod_{v_0 \in V_0} \mathcal{I}^{\mathbb{Q}}({}^{S^{\pm}_{j+1};\, n,\circ}\mathcal{D}^{\vdash}_{v_0})$$

w/

$$\mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{MOD}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}})_j \overset{\sim}{\to} \mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}})_j$$

(global can be calc'd by local log-vol's)

${}^{n,\circ}\mathcal{R}^{\mathrm{LGP}}$: the above data (a), (b), (c)
up to the following indet.

(Indet $\curvearrowright$) $\mathrm{Prc}({}^{n,\circ}\mathfrak{S})^{\vdash}_{T}$ $\leftarrow$ up to autom.

(Indet $\longrightarrow$) $v_\Theta \in \mathbb{V}_\Theta^{\mathrm{non}}$ (arc omit)

$\mathcal{I}^{\Theta}({}^{\pm}\mathfrak{S}_{j+1};{}^{n,\circ}\mathfrak{D}^{\vdash}_{v_\Theta})$

$\Theta \ldots \Theta$

$j+1$

up to Isomet on each pieces indep.'ly.

$(\mathbb{P}_1 \subset \mathbb{P}_2 \subset \cdots \subset \mathbb{P}_n)$
all capsule-full poly-morph

$(\Theta_1 \subset \cdots \subset \Theta_m)$
morph order-preserving inj
$\iota: \{1,\ldots,n\} \hookrightarrow \{1,\ldots,m\}$
w/ capsule-full poly
$\mathbb{P}_j \subset \Theta_{\iota(j)}$
$j = 1, \ldots, n$

$\sim$ ${}^{n,\circ}\mathcal{R}^{\mathrm{LGP}}$
full poly

permutation symm. of
étale picture

$\sim$ $\mathrm{Prc}({}^{n,\circ}\mathfrak{S})^{\vdash}_{T} \xrightarrow[\sim]{\text{poly}} \mathrm{Prc}({}^{n',\circ}\mathfrak{S})^{\vdash}_{T}$

${}^{n,\circ}\mathcal{R}^{\mathrm{LGP}} \underset{\text{poly}}{\xrightarrow{\sim}} {}^{n',\circ}\mathcal{R}^{\mathrm{LGP}}$ compat. w/

${}^{n,\circ}\mathfrak{S}^{\vdash}_{\Theta} \underset{\text{poly}}{\xrightarrow{\sim}} {}^{n',\circ}\mathfrak{S}^{\vdash}_{\Theta}$ $\leftarrow$ Θ-label

bijectivity

(ii) (log-Kummer corresp.)

$n, m \in \mathbb{Z}$ Km ...

(ii) (log-Kummer corresp.)

$n, m \in \mathbb{Z}$

induces

$$\begin{cases} \mathrm{Kmm} : \mathcal{I}_{\mathrm{cns}}({}^{n,m}\mathcal{F}_{v})|_{*} \xrightarrow{\sim} \mathcal{I}_{\mathrm{cns}}({}^{n,\circ}\mathcal{F}_{v})|_{*} \quad \text{Frob, et, rad.-cnic} \\ \mathrm{Kmm} : \{\pi_1^{\mathrm{rat}}({}^{n,m}\mathcal{D}^{\circledast}) \curvearrowright {}^{n,m}\mathbb{M}^{\circledast}_{\infty\kappa}\}_j \xrightarrow{\sim} \{\pi_1^{\mathrm{rat}}({}^{n,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledast}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc})\}_j \\ \mathrm{Kmm} : ({}^{n,m}\overline{\mathbb{M}}^{\circledcirc}_{\mathrm{mod}})_j \xrightarrow{\sim} \overline{\mathbb{M}}^{\circledcirc}_{\mathrm{mod}}({}^{n,\circ}\mathcal{D}^{\circledcirc})_j \end{cases}$$

(a). $\underline{\mathbb{V}} \ni \underline{v} \in \underline{\mathbb{V}}^{\mathrm{non}}$, $j \in |\mathbb{F}_l|$

$$\mathcal{I}^{(\circledcirc)}(\mathcal{S}^{\pm}_{j+1};{}^{m,n}\mathcal{F}_{v_\circledcirc}) \xrightarrow{\sim} \mathcal{I}^{(\circledcirc)}(\mathcal{S}^{\pm}_{j+1};{}^{n,m}\mathcal{F}^{\vdash \times \mu}_{v_\circledcirc}) \xrightarrow[\text{Ism-orbit}]{\sim\ \mathrm{poly}} \mathcal{I}^{(\circledcirc)}(\mathcal{S}^{\pm}_{j+1};{}^{n,\circ}\mathcal{D}^{\vdash}_{v_\circledcirc})$$

$\mathcal{S}^{\pm}_{j+1};j(\ )_{\underline{v}}$ $\quad$ $\mathcal{S}^{\pm}_{j+1};j(\ )_{\underline{v}}$ $\quad$ $\mathcal{S}^{\pm}_{j+1};j(\ )_{\underline{v}}$

compat w/ log-vol.

(b) $\underline{\mathbb{V}}^{\mathrm{bad}} \ni \underline{v}$ splitting monoid

const. mult. rig.

$$\mathcal{I}^{\perp}_{\mathcal{F}_{\mathrm{LGP}}}({}^{n,m}\mathcal{HT}^{\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_{\underline{v}} \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{I}^{\perp}_{\mathcal{F}_{\mathrm{LGP}}}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_{\underline{v}}$$

Frob. $\quad$ et

(c) $j \in \mathbb{F}_l^{*}$

$F^{\times}_{\mathrm{mod}} \cap \prod_{\underline{v}} \mathcal{O}_{\underline{v}} = \mu$

NF l rat'l'd / non-rat'l'd gl. Frob'd

$m \in \mathbb{Z}$ mutually compat. ($\leftarrow$ non-interference)

$$({}^{n,m}\overline{\mathbb{M}}_{\mathrm{MOD}\ \mathrm{mod}})_j \xrightarrow[\mathrm{Kmm}]{\sim} \overline{\mathbb{M}}_{\mathrm{MOD}\ \mathrm{mod}}^{\circledcirc}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_j \quad \leftarrow \text{MOD-side}$$

$$({}^{n,m}\mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}\ \mathrm{mod}})_j \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}\ \mathrm{mod}}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_j \quad \mathbb{R}\text{-version}$$

Frob $\quad$ et (rat. cnic)

$${}^{n,m}\mathcal{C}^{\Vdash}_{\mathrm{LGP}} \xrightarrow{\sim} \mathcal{C}^{\Vdash}_{\mathrm{LGP}}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}}) \quad \mathcal{F}^{\Vdash}({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})$$

Frob, lgp, et, compat. $\quad {}^{n,m}\mathcal{F}^{\Vdash}_{\mathrm{LGP},\ \mathrm{lgp}}$

We consider the above data (a) up to the following indet.

$\mathcal{I}_{\underline{v}}$ $\quad$ $\mathcal{I}^{\mathbb{Q}}$ $\quad$ $|F^{\times}_{\mathrm{mod}}|_{\underline{v}}$

$m \in \mathbb{Z}$, isom (a) is up ... compat.

[Indet ↑]

($\circledcirc^{\times\mu}_{\mathrm{LGP}}$-link compatibility)

Kmm's in (ii) have the foll... w.r.t.

We consider the above data (a) up to the following indet.

(Indet ↑) $m \in \mathbb{Z}$, isom (a) is upper semi-compact.

(iii) ($\Theta^{\times\mu}_{\mathrm{LGP}}$-link compatibility)

Kmm's in (ii) have the following properties w.r.t. $\Theta^{\times\mu}_{\mathrm{LGP}}$-link

(a). Kmm in (ii) $\xrightarrow[\mathrm{F}_l^{\times\pm}\text{-symm}]{}$ ${}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \xrightarrow{\sim} \sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,0}\varsigma)^{\vdash}_{\Delta}$

rel. to

$\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,0}\varsigma)^{\vdash}_{\Delta}) \xrightarrow[\text{full poly}]{\sim} \sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n+1,0}\varsigma)^{\vdash}_{\Delta})$ of $\mathcal{F}^{\vdash\times\mu}$-prime-strip

is compat. w/

$\mathcal{I}^{\mathbb{Q}}$ $(\mathcal{I}^{j^2})$ $(F^{\times}_{\mathrm{mod}})_j$

$${}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \xrightarrow[\text{full poly}]{\sim} {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}$$

$\Theta^{\times\mu}_{\mathrm{LGP}}$-link

$$\begin{array}{ccc}
{}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} & \xrightarrow[\text{full poly}]{\sim} & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \\
\downarrow \wr\, \mathrm{Kmm} & \circlearrowleft & \downarrow \wr\, \mathrm{Kmm} \\
\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,0}\varsigma)^{\vdash}_{\Delta}) & \xrightarrow[\text{full poly}]{\sim} & \sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n+1,0}\varsigma)^{\vdash}_{\Delta})
\end{array}$$

(b). $\mathcal{F}^{\Vdash}$-prime-strips ${}^{n,m}\sigma\mathcal{F}^{\Vdash}_{\mathrm{env}}$, $\sigma\mathcal{F}^{\Vdash}_{\mathrm{env}}({}^{n,0}\varsigma)_{>})$

$\rightsquigarrow$ $\mathcal{F}^{\vdash\times\mu}$-prime-strip isom

$${}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \xrightarrow{\sim} {}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\mathrm{env}}$$

$$\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,0}\varsigma)^{\vdash}_{\Delta}) \xrightarrow{\sim} \sigma\mathcal{F}^{\vdash\times\mu}_{\mathrm{env}}({}^{n,0}\varsigma)_{>})$$

$$\begin{array}{ccc}
{}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} & \xrightarrow[\text{full poly}]{\overset{\Theta\text{-link}}{\sim}} & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \\
\mathrm{Kmm} + \Delta\to\mathrm{env}\ \downarrow \wr & \circlearrowleft & \downarrow \wr\ \Delta\ \mathrm{Kmm} + \Delta\to\mathrm{env} \\
\sigma\mathcal{F}^{\vdash\times\mu}_{\mathrm{env}}({}^{n,0}\varsigma)^{\vdash}_{\Delta}) & \xrightarrow[\text{full poly}]{\sim} & \sigma\mathcal{F}^{\vdash\times\mu}_{\mathrm{env}}({}^{n+1,0}\varsigma)^{\vdash}_{\Delta})
\end{array}$$

Summary of Th 3.11

(i) ("protagonists")

unit portion $\rightsquigarrow$ $\mathcal{I}$

⊞ $\mathbb{F}_l^{\rtimes \pm}$ mul. ?P $\mathcal{F}^{+}$ LGP log-link

compat of log-link w/ $\mathbb{F}_l^{\rtimes\pm}$-symm.

⊠ $\mathbb{F}_l^{*}$ NF $\overline{M}_{\mathrm{mod}}$ ← Belyi cuspidalization (hidden endom + pGC)

(ii) (log-Kummer)

upper semi-compat. $\mathcal{O}^\times \subset \frac{1}{2p}\log(\mathcal{O}^\times)$

(Indet ↑)

non-interference

const, mult, rig. ← ell. cuspidalization (hidden endom + pGC)

non-interference

$F_{\mathrm{mod}} \cap \prod_v \mathcal{O} = \mu$

(iii) (compat w/ $\Theta^{\times\mu}_{LGP}$-link) ← $\mathbb{F}_l^{\rtimes\pm}$-conj. synchro

inv. up to (Indet →)

$\hat{\mathbb{Z}}^\times$-indet ← harmless for log-vol.

protected from $\hat{\mathbb{Z}}^\times$-indet

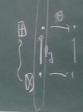
mono-theta cycl. rig. (← ?)

protected from $\hat{\mathbut{Z}}^\times$-indet.

$\mathbb{Q}_{>0} \cap \hat{\mathbb{Z}}^\times = \{1\}$

inv. étale picture up to (Indet ?)

temporal conj vs proof conj (IUT ~ temporal ...)



$\mathbb{V}_{\mathbb{Q}} \supset V$ fin. subset $\supset \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}}$

$V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}} \ni v$ (resp. $V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}} \ni v$)

$\phi \neq \mathcal{K}_v \subsetneq X^{\mathrm{arc}}$ (resp. $\phi \neq \mathcal{K}_v \subsetneq X(\overline{\mathbb{Q}}_v)$)

$X(\mathbb{C})$

a $\mathrm{Gal}(\mathbb{C}/\mathbb{R})$-stable cpt domain

(resp. a $\mathrm{Gal}(\overline{\mathbb{Q}}_v/\mathbb{Q}_v)$-stable subset s.t. $\forall [K:\mathbb{Q}_v] < \infty$, $X(K) \cap \mathcal{K}_v$: cpt domain)

cpt domain ← cpt subset of top. space s.t. it is the closure of its interior

$\mathcal{K}_V \subset X(\overline{\mathbb{Q}})$

$\{x \in X(\overline{\mathbb{Q}}) \mid x \in X(F)$, $F/\mathbb{Q}$ fin., $\forall v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}}$ (resp. $v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}}$) $\}$

$\mathbb{V}_{\mathbb{Q}} \supset V$ fin. subset $\supset \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}}$

$V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}} \ni v$ (resp. $V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}} \ni v$)

$$\emptyset \neq \mathcal{K}_v \subsetneq X^{\mathrm{arc}} \quad \left(\text{resp. } \emptyset \neq \mathcal{K}_v \subsetneq X(\overline{\mathbb{Q}}_v)\right)$$

$X^{\mathrm{arc}} \wr X(\mathbb{C})$

a $\mathrm{Gal}(\mathbb{C}/\mathbb{R})$-stable cpt domain

(resp. a $\mathrm{Gal}(\overline{\mathbb{Q}}_v/\mathbb{Q}_v)$-stable subset s.t. $\forall [K:\mathbb{Q}_v] < \infty$, $X(K) \cap \mathcal{K}_v$: cpt domain)

cpt domain: cpt subset of top. space s.t. it is the closure of its interior

$\mathcal{K}_V \subset X(\overline{\mathbb{Q}})$ ← compactly bdd subset

$$\mathcal{K}_V := \left\{ x \in X(\overline{\mathbb{Q}}) \;\middle|\; \begin{array}{l} x \in X(F),\ F/\mathbb{Q} \text{ fin.} \\ \forall v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}} \ (\text{resp. } v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}}) \\ \text{the set of } [F:\mathbb{Q}] \text{ pts of } X^{\mathrm{arc}} \\ (\text{resp. } X(\overline{\mathbb{Q}}_v)) \text{ det'd by } x \\ \in \mathcal{K}_v \end{array} \right\}$$

$\mathcal{K}_V \leadsto V, \mathcal{K}_v$

support

Prop 1.2 ([GenEll, Th 2.1])

The following ①, ② are equiv.

① **Th 0.1** (Vojta's conjecture for curves (S. Mochizuki))

X : proper smooth geom. conn. curve / a NF

$X \supset D$: red. div. $U_X := X \setminus D$, ω_X : can. sheaf on X

U_X : hyperbolic (i.e. $\deg(\omega_X(D)) > 0$) $\forall d \in \mathbb{Z}_{>0}$, $\forall \varepsilon \in \mathbb{R}_{>0}$

we have

$$ht_{\omega_X(D)} \lesssim (1+\varepsilon)(\text{log-diff}_X + \text{log-cond}_D)$$

on $U_X(\overline{\mathbb{Q}})^{\leq d}$

② fix Σ : fin. set of prime

$U_{\mathbb{P}^1} := \mathbb{P}^1_{\mathbb{Q}} \setminus \{0, 1, \infty\}$

$\mathcal{K}_V \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})$: a cptly bdd subset whose support $\supset \Sigma$

$\forall d \in \mathbb{Z}_{>0}$, $\forall \varepsilon \in \mathbb{R}_{>0}$

$$ht_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})} \lesssim (1+\varepsilon)(\text{log-diff}_{\mathbb{P}^1} + \text{log-cond}_{\{0,1,\infty\}})$$

on $\mathcal{K}_V \cap U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$

$\mathbb{V}_{\mathbb{Q}} \supset V$ fin. subset $\supset \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}}$

$V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}} \ni v$ (resp. $V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}} \ni v$)

$\phi \neq \mathcal{K}_v \subsetneq X^{\mathrm{arc}}$ (resp. $\phi \neq \mathcal{K}_v \subsetneq X(\overline{\mathbb{Q}}_v)$)

$X^{\mathrm{arc}} \wr X(\mathbb{C})$

a $Gal(\mathbb{C}/\mathbb{R})$-stable cpt domain

(resp. a $Gal(\overline{\mathbb{Q}}_v/\mathbb{Q}_v)$-stable subset s.t. $\forall [K:\mathbb{Q}_v] < \infty$, $X(K) \cap \mathcal{K}_v$: cpt domain)

cpt domain: cpt subset of top. space s.t. it is the closure of its interior

$\mathcal{K}_V \subset X(\overline{\mathbb{Q}})$ ← compactly bdd subset

$$\mathcal{K}_V := \left\{ x \in X(\overline{\mathbb{Q}}) \;\middle|\; \begin{array}{l} x \in X(F),\ F/\mathbb{Q} \text{ fin.} \\ \forall v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}} \text{ (resp. } v \in V \cap \mathbb{V}_{\mathbb{Q}}^{\mathrm{non}}) \\ \text{the set of } [F:\mathbb{Q}] \text{ pts of } X^{\mathrm{arc}} \\ \text{(resp. } X(\overline{\mathbb{Q}}_v)) \text{ det'd by } x \\ \in \mathcal{K}_v \end{array} \right\}$$

$\mathcal{K}_V \rightsquigarrow V$, $\mathcal{K}_v$

V: support.

<u>Prop 1.2</u> ([GenEll, Th 2.1])

The following ①, ② are equiv.

① <u>Th 0.1</u> (Vojta's conjecture for curves (S. Mochizuki))

X : proper smooth geom. conn. curve / a NF

$X \supset D$: red. div., $U_X := X \setminus D$, ω_X : can. sheaf on X

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we have

$$ht_{\omega_X(D)} \underset{\sim}{<} (1+\varepsilon)(\text{log-diff}_X + \text{log-cond}_D)$$

on $U_X(\overline{\mathbb{Q}})^{\leq d}$

$U_X(\overline{\mathbb{Q}}) = \bigcup_{F/\mathbb{Q} \text{ fin}} U_X(F)$

$\cup$

$U_X(\overline{\mathbb{Q}})^{\leq d} := \bigcup_{[F:\mathbb{Q}] \leq d} U_X(F)$

② fix Σ : fin. set of prime

$U_{\mathbb{P}^1} := \mathbb{P}^1_{\mathbb{Q}} \setminus \{0, 1, \infty\}$

$\mathcal{K}_V \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})$: a cpt'ly bdd subset whose support $\supset \Sigma$

$\forall d \in \mathbb{Z}_{>0}$, $\forall \varepsilon \in \mathbb{R}_{>0}$

$$ht_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})} \underset{\sim}{\leq} (1+\varepsilon)(\text{log-diff}_{\mathbb{P}^1} + \text{log-cond}_{\{0,1,\infty\}})$$

on $\mathcal{K}_V \cap U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$

① ⇒ ② special case

① ⇐ ② non-crit. Belyi maps [Belyi]

<u>omit</u>

② Σ : fix fin. set of prime

$U_{\mathbb{P}^1} := \mathbb{P}^1_{\mathbb{Q}} \setminus \{0, 1, \infty\}$

$K_V \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})$: a cpt'ly bdd subset whose support $\supset \Sigma$

$\forall d \in \mathbb{Z}_{>0}$, $\forall \varepsilon \in \mathbb{R}_{>0}$

$$\mathrm{ht}_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})} \lesssim (1+\varepsilon)(\text{log-diff}_{\mathbb{P}^1} + \text{log-cond}_{\{0,1,\infty\}})$$

on $K_V \cap U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$

① $\Rightarrow$ ② special case

① $\Leftarrow$ ② non-crit. Belyi maps [Belyi] & log-diff. behavior of coverings !

omit

Lem 1.3 ([IUTch IV, Prop 1.2 (i)])

$k/\mathbb{Q}_p$ fin. e : ram. index of $k/\mathbb{Q}_p$

$\lambda \in \frac{1}{e}\mathbb{Z}$, $p^{\lambda}\mathcal{O}_k := (\varpi)^{e\lambda}$ (unif)

$$a := \begin{cases} \frac{1}{e}\left\lceil \frac{e}{p-2} \right\rceil & p > 2 \\ 2 & p = 2 \end{cases} \qquad b := \left\lfloor \frac{\log\left(p\frac{e}{p-1}\right)}{\log p} \right\rfloor - \frac{1}{e}$$

$$\Rightarrow p^{a}\mathcal{O}_k \subseteq \log_p(\mathcal{O}_k^{\times}) \subseteq p^{-b}\mathcal{O}_k$$

If $p > 2$, $e \leq p-2$

$\Rightarrow p^{\lambda}\mathcal{O}_k = \log_p(\mathcal{O}_k^{\times}) = p^{-b}\mathcal{O}_k$

(omit)

$-|\log(\underline{\Theta})| \leq 0$

Prop 1.12 ([IUTch IV, Th 1.10])

initial theta data ⓔ

q-param. at bad fin. V

$$-|\log(\underline{q})| = -\frac{1}{2\ell}\log(\mathfrak{q})$$

$\mathbb{P}^1 \setminus \{0,1,\infty\}$ λ-line

$$-|\log(\underline{\Theta})| \leq -\frac{1}{2\ell}\log(\mathfrak{q}) + \frac{\ell+1}{4}\left(-\frac{1}{6}\left(1-\frac{12}{\ell^2}\right)\log(\mathfrak{q}) + \left(1+\frac{36\, d_{\mathrm{mod}}}{\ell}\right)\left(\text{log-diff}_{\mathbb{P}^1}(x_E) + \text{log-cond}_{\{0,1,\infty\}}(x_E)\right) + 10\left(d^{*}_{\mathrm{mod}}\, \ell + \eta_{\mathrm{prm}}\right)\right)$$

ht

const.

prime number theorem

$$\rotatebox{90}{$\leq$} \quad -|\log(\underline{q})| = -\frac{1}{2\ell}\log(\mathfrak{q})$$

$d_{\mathrm{mod}} := [F_{\mathrm{mod}} : \mathbb{Q}]$

$d^{*}_{\mathrm{mod}} := d_{\mathrm{mod}} \cdot \underbrace{2^{12} \cdot 3^{3} \cdot 5}_{2 \times \#(\mathbb{Z}/4)^{\times} \cdot \#GL_2(\mathbb{F}_2)\, \#GL_2(\mathbb{F}_3)\, \#GL_2(\mathbb{F}_5)}$

IUTch $\underbrace{-|\log(\underline{q})|}_{0} \leq -|\log(\underline{\Theta})|$

$|\log(\underline{\Theta})| \leq 0$ ⇝ NF hyperbolicity

$\Delta \subset \mathbb{Z} \times_{\mathbb{F}_1} \mathbb{Z}$; hol. hull

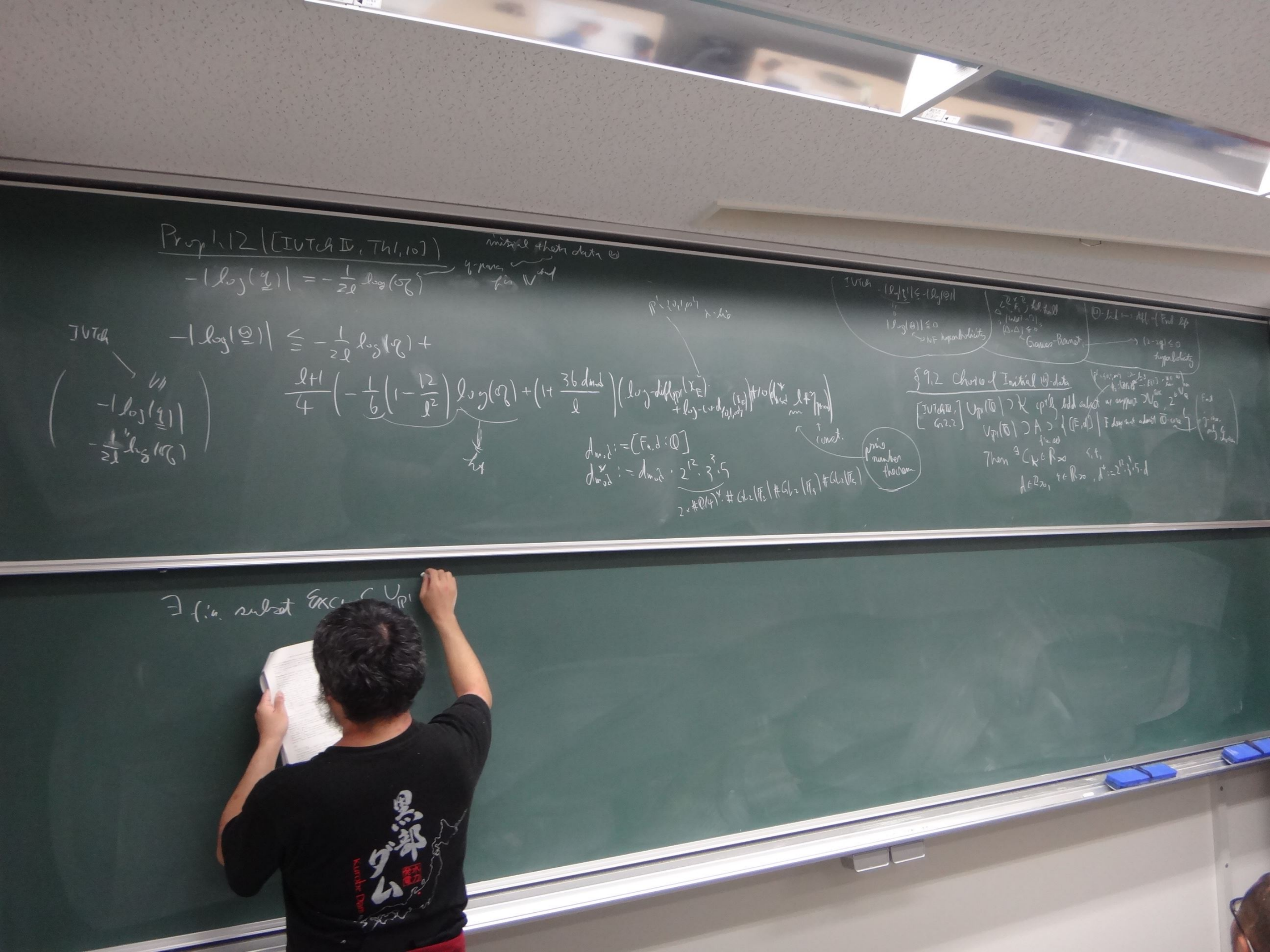
"(ind 1 - 2)"

"$(\Delta \cdot \Delta) \leq 0$" ← Gauss-Bonnet

Θ-link ⟼ Aff. of Frob. lift

⇝ $(2-2g) \leq 0$ hyperbolicity

§9.2 Choice of Initial Θ-data



Prop 1.12 ([IUTch IV, Th 1.10])
initial theta data
q-para
IUTch
prime number theorem
const.
§9.2 Choice of Initial Θ-data
Gauss-Bonnet
NF hyperbolicity
hyperbolicity
Then
∃ fin. subset
黒部ダム

$\exists$ fin. subset $\mathrm{Exc}_{K,d,\varepsilon} \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$ s.t.

$\mathrm{Exc}_{K,d,\varepsilon} \supset A$

$x = [(E_F, \alpha)] \in (U_{\mathbb{P}^1} \cap K) \setminus \mathrm{Exc}_{K,d,\varepsilon}$ w/ $[F:\mathbb{Q}] \leq d$

F_{mod} : field of moduli of $E_{\overline{F}} := E_F \times_F \overline{F}$

$F_{\mathrm{tpd}} := F_{\mathrm{mod}}(E_{F_{\mathrm{mod}}}[2]) \subset F$

Assume all pts of $E_F[3 \cdot 5]$ rat. $/F$

$(\& F = F_{\mathrm{tpd}}(\sqrt{-1}, E_{F_{\mathrm{tpd}}}[3 \cdot 5]))$

$\Rightarrow$ E_F, F_{mod} arise as a part of initial Θ-data

& $\mathrm{ht}_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})}(x) \leq (1+\varepsilon)\begin{pmatrix} \text{log-diff}_{\mathbb{P}^1}(x) \\ + \text{log-cond}_{\{0,1,\infty\}}(x) \end{pmatrix} + C_K$

pf)

$\mathrm{Exc}_{K,d} := A$ ~ we will enlarge in the proof

$x = [(E_F, \alpha)] \in U_{\mathbb{P}^1}(F) \cap K \setminus \mathrm{Exc}_{K,d}$

$\vartheta(x) := \sum_{\substack{p \leq x \\ \text{prime}}} \log p \sim x \quad (x \to \infty)$

(Chebyshev's θ-fct) (↑ prime number thm)

Take $\exists \xi_{\mathrm{prm}} \geq 5$ s.t.

$\frac{2}{3}x < \vartheta(x) \leq \frac{4}{3}x$ for $\forall x \geq \xi_{\mathrm{prm}}$

$h := h(E_F) = \log(\mathfrak{q}^{\forall}) = \frac{1}{[F:\mathbb{Q}]} \sum_{v \in V(F)^{\mathrm{non}}} h_v f_v \log(p_v)$

$\frac{1}{6}\log(\mathfrak{q}^{\forall}) \approx \mathrm{ht}_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})}$ (deg. of res. field $\mathbb{F}_{p_v}$)

$(h_v = 0 \Leftrightarrow E_F$ has good red. at $v)$

$\exists$ fin. many isom. classes of E_F w/ $h^{\frac{1}{2}} < \xi_{\mathrm{prm}} + \eta_{\mathrm{prm}}$

Enlarge $\mathrm{Exc}_{K,d}$, we may assume

(s1) $h^{\frac{1}{2}} \geq \xi_{\mathrm{prm}} + \eta_{\mathrm{prm}}$

$(\xi$

$$\left(\xi_{prm} \geq 5,\ \eta_{prm} > 0 \Rightarrow h^{\frac{1}{2}} \geq 5 \right)$$

$$(s2) \quad 2d^* h^{\frac{1}{2}} \log(2d^* h) \geq 2[F:\mathbb{Q}] h^{\frac{1}{2}} \log(2[F:\mathbb{Q}] h)$$
$$\geq \sum_{h_n \neq 0} 2 h^{-\frac{1}{2}} \log(2 h_n f_n \log(p_n)) h_n f_n \log(p_n)$$
$$\geq \sum_{h_n \neq 0} h^{-\frac{1}{2}} \log(h_n) h_n \geq \sum_{h_n \geq h^{\frac{1}{2}}} h^{-\frac{1}{2}} \log(h_n) h_n \geq \sum_{h_n \geq h^{\frac{1}{2}}} \log(h_n)$$

$$(s3) \quad d^* h^{\frac{1}{2}} \geq [F:\mathbb{Q}] h^{\frac{1}{2}} = \sum_{n:\mathrm{non}} h^{-\frac{1}{2}} h_n f_n \log(p_n) \geq \sum_{n:\mathrm{non}} h^{-\frac{1}{2}} h_n \log(p_n)$$
$$\geq \sum_{h_n \geq h^{\frac{1}{2}}} h^{-\frac{1}{2}} h_n \log(p_n) \geq \sum_{h_n \geq h^{\frac{1}{2}}} \log(p_n)$$

$\mathcal{A} \subset$ Primes

$\parallel$ $\{p : $ satisfies $(\mathfrak{s}1)$ or $(\mathfrak{s}2)$ or $(\mathfrak{s}3)\}$

$(\mathfrak{s}1)$ $p \leq h^{\frac{1}{2}}$

$(\mathfrak{s}2)$ $p \mid h_n \neq 0$ for some $n \in \mathbb{V}(F)^{non}$

$(\mathfrak{s}3)$ $p = p_n$ for some $n \in \mathbb{V}(F)^{non}$ & $h_n \geq h^{\frac{1}{2}}$

$\Rightarrow$ $(\mathfrak{s}'1)$ $\sum_{p:(\mathfrak{s}1)} \log(p) = \vartheta(h^{\frac{1}{2}}) \leq \frac{4}{3} h^{\frac{1}{2}}$ (by 2nd $\leq$ of $(s0)$ & $h^{\frac{1}{2}} \geq \xi_{prm}$ $(\Leftarrow (s1))$)

$(\mathfrak{s}'2)$ $\sum_{\substack{p:(\mathfrak{s}2) \\ \mathrm{not}\ (\mathfrak{s}3)}} \log(p) \leq \sum_{h_n \geq h^{\frac{1}{2}}} \log(h_n) \leq 2d^* h^{\frac{1}{2}} \log(2d^* h)$ by $(s2)$

$(\mathfrak{s}'3)$ $\sum_{p:(\mathfrak{s}3)} \log(p) \leq d^* h^{\frac{1}{2}}$ by $(s3)$

$$\Rightarrow (\mathfrak{s}'123) \quad \vartheta_{\mathcal{A}} := \sum_{p \in \mathcal{A}} \log(p)$$
$$\leq 2 h^{\frac{1}{2}} + d^* h^{\frac{1}{2}} + 2d^* h^{\frac{1}{2}} \log(2d^* h)$$
$$\leq 4 d^* h^{\frac{1}{2}} \log(2d^* h) \leq -\xi_{prm} + 5 d^* h^{\frac{1}{2}} \log(2d^* h)$$

$\underset{(s1)}{}$

$(\mathfrak{s}'1), (\mathfrak{s}'2), (\mathfrak{s}'3)$ $\quad 2h^{\frac{1}{2}} \leq d^* h^{\frac{1}{2}}$ & $\log(2d^* h^{\frac{1}{2}}) \geq \log 4 > 1$

$\rightsquigarrow$ $\exists\, l \notin \mathcal{A}$ prime s.t. $l \leq 2(\vartheta_{\mathcal{A}} + \xi_{prm})$

(☹ otherwise $\vartheta_{\mathcal{A}} \geq \vartheta(2(\vartheta_{\mathcal{A}} + \xi_{prm})) \geq \frac{2}{3}(2(\vartheta_{\mathcal{A}} + \xi_{prm})) \geq \frac{4}{3} \vartheta_{\mathcal{A}}$ by 2nd $\leq$ of $(s0)$

contrad.)

$l \notin \mathcal{A}$

$\Rightarrow$ (P1) (upper b[…]d of l)

$(s\leq)$ $h^{\frac{1}{2}}$ […] $\leq 10 d^* h^{\frac{1}{2}} \log(2d^* h)$

l not […] $(\mathfrak{s}1)$

$\Rightarrow$ (S'123) $\vartheta_{\mathcal{A}} := \sum_{p \in \mathcal{A}} \log(p)$

$\leq 2h^{\frac{1}{2}} + d^* h^{\frac{1}{2}} + 2d^* h^{\frac{1}{2}} \log(2d^* h)$

$\leq 4 d^* h^{\frac{1}{2}} \log(2d^* h) \leq -\xi_{prm} + 5 d^* h^{\frac{1}{2}} \log(2d^* h)$

(S'1), (S'2), (S'3) $\qquad$ (S1)

$2h^{\frac{1}{2}} \leq d^* h^{\frac{1}{2}}$ & $\log(2d^* h^{\frac{1}{2}}) \geq \log(4) > 1$

$\rightsquigarrow \exists\, l \notin \mathcal{A}$ prime s.t. $l \leq 2(\vartheta_{\mathcal{A}} + \xi_{prm})$

(☹ otherwise $\vartheta_{\mathcal{A}} \geq \vartheta(2(\vartheta_{\mathcal{A}} + \xi_{prm})) \geq \frac{2}{3}(2(\vartheta_{\mathcal{A}} + \xi_{prm})) \geq \frac{4}{3}\vartheta_{\mathcal{A}}$

by 2nd $\leq$ of (S0)

contrad.)

$l \notin \mathcal{A}$

$\Rightarrow$ (P1) (upper bound of l)

(S$\leq$) $h^{\frac{1}{2}} < l \leq 10 d^* h^{\frac{1}{2}} \log(2d^* h)$ $\quad (\leq 20 (d^*)^2 h^2)$

l not satis. (S1) $\qquad l \leq 2(\vartheta_{\mathcal{A}} + \xi_{prm})$ & (S'123)

$\log(2d^* h) \leq 2d^* h \leq 2d^* h^{\frac{3}{2}}$ $\quad \left(\begin{array}{l}\log x \leq x \\ h \geq 1\end{array}\right)$

(P2) (monodromy non-vanishing modulo l)

$l \nmid h_v$ for $\forall v \in \mathbb{V}(F)^{\text{bad}}$ s.t. $h_v \neq 0$

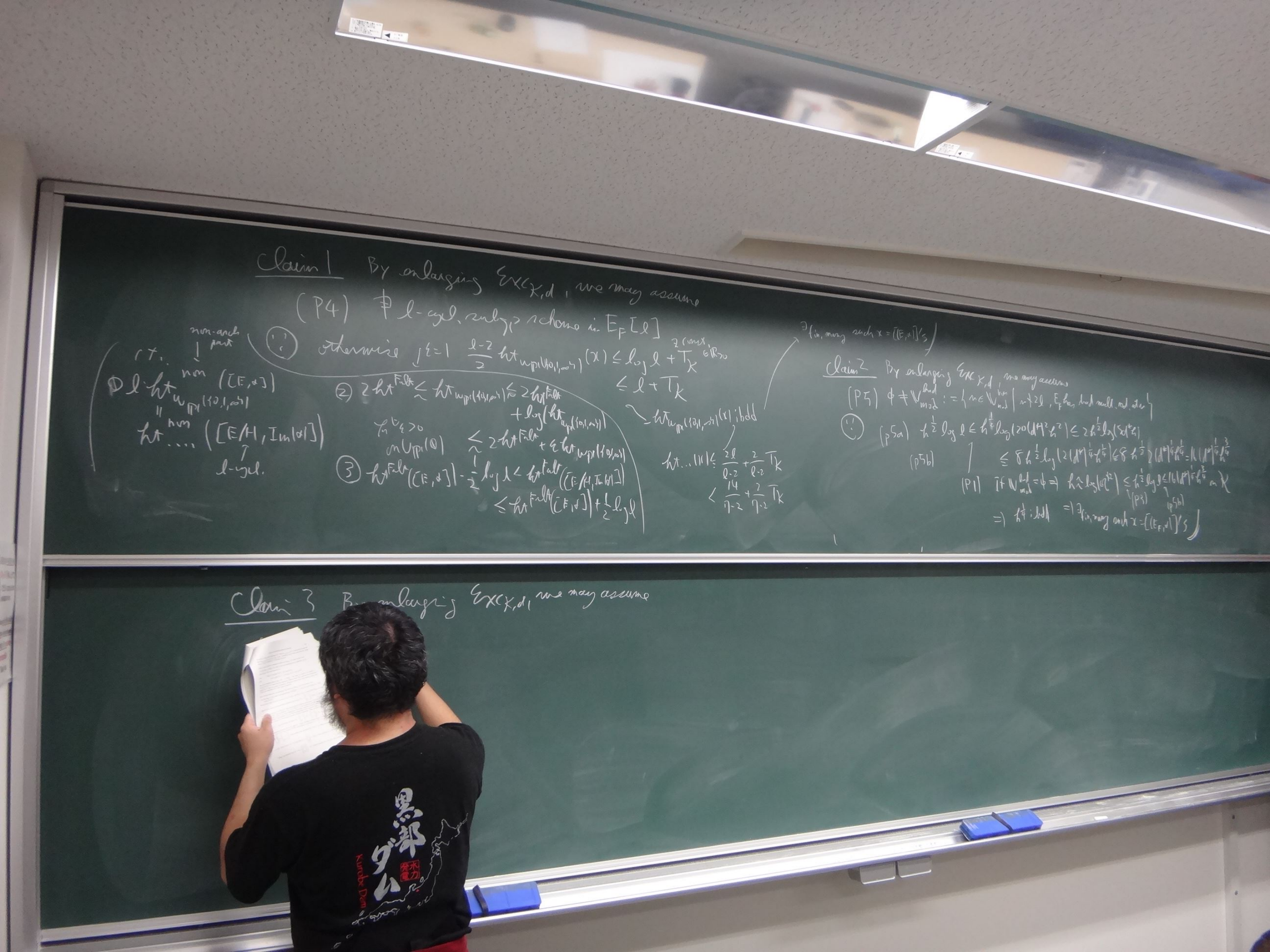
l not satis. (S2)

(P3) (upper bound of monodromy at l)

If $l = p_v$ for some $v \in \mathbb{V}(F)^{\text{bad}}$ $\Rightarrow h_v < h^{\frac{1}{2}}$

l not satis. (S3)

claim 1



Claim 1 By enlarging Exc_{K,d}, we may assume
(P4) ∄ ℓ-cycl. subgrp scheme in E_F[ℓ]
otherwise
non-arch part
ℓ · ht^{nm}_{w_{P^1}(\{0,1,∞\})}([E, α]) = ht^{nm}_{....}([E/H, Im(α)])
ℓ-cycl.
ht^{Falt} ≲ ht_{w_{P^1}(\{0,1,∞\})} ≲ 2 ht^{Falt} + log(ht_{w_{P^1}(\{0,1,∞\})})
ht^{Falt}([E, α]) − 1/2 log ℓ < ht^{Falt}([E/H, Im(α)]) ≤ ht^{Falt}([E, α]) + 1/2 log ℓ
(ℓ−2)/2 ht_{w_{P^1}(\{0,1,∞\})}(x) ≤ log ℓ + T_K ≤ ℓ + T_K
⇝ ht_{w_{P^1}(\{0,1,∞\})}(x): bdd
ht...(x) ≤ 2ℓ/(ℓ−2) + 2/(ℓ−2) T_K < 14/(7−2) + 2/(7−2) T_K
∃ fin. many such x = [(E, α)]'s
Claim 2 By enlarging Exc_{K,d}, we may assume
(P5) ∅ ≠ V^{bad}_{mod}
(P5a)
(P5b)
(P1) if V^{bad}_{mod} = ∅ ⇒
(P3)
(P5b)
⇒ h^{1/4}: bdd
⇒ ∃ fin. many such x = [(E_F, α)]'s
Claim 3 By enlarging Exc_{K,d}, we may assume

Claim 3 By enlarging $\mathrm{Exc}_{\mathcal{X},d}$, we may assume

(P6) The image of the outer hom

$$\mathrm{Gal}(\overline{\mathbb{Q}}/F) \to GL_2(\mathbb{F}_\ell)$$

$E[\ell]$ contains $SL_2(\mathbb{F}_\ell)$

☺ (cf. [GenEll, Lem 3.1 (i), (iii)])

(P2) $\Rightarrow \ell \nmid h_\infty \neq 0$ (P5) $\Rightarrow \mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}} \neq \phi$

the image $H \ni N_+ := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

generates an ℓ-Sylow subgp S of $GL_2(\mathbb{F}_\ell)$

$\#\{\ell\text{-Sylow subgp of } GL_2(\mathbb{F}_\ell)\} = \ell+1$

$N_{GL_2(\mathbb{F}_\ell)}(S) = \left\{\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}\right\}$

(P4) ($E[\ell]$ $\nmid$ (lugsl.))

$\Rightarrow H \ni$ a matrix : not upper triangular

$\Rightarrow n_H := \#\{\ell\text{-Sylow in } H\} > 1$

$n_H \equiv 1 \pmod{\ell} \Rightarrow n_H = \ell+1$ ($\Leftarrow 1 < n_H \leq \ell+1$)

$\Rightarrow N_+, N_- := \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \in H$.

$SL_2(\mathbb{F}_\ell) \underset{\text{easy}}{=} G := \langle N_+, N_- \rangle$ ⌟

$E_F, \overline{F}, \ell, \mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}}$

(P7) $\exists \underline{C}_K, \exists \underline{\mathbb{V}}, \exists \underline{\epsilon}$ s.t.

$(\overline{F}/F, X_F, \ell, \underline{C}_K, \underline{\mathbb{V}}, \mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}}, \underline{\epsilon})$

is an initial Θ-data

☺ (P6)

$\rightsquigarrow$ Apply $\boxed{-|\log(\underline{\underline{q}})| \leq -|\log(\underline{\underline{\Theta}})|}$ IUTch

(A) $\frac{1}{6}\log(\eta) \le \left(1+\frac{80 d_{mod}}{\ell}\right)\left(\log(\delta^{F+pd}) + \log(f^{F+pd})\right) + 20(d^*_{mod}\,\ell + \eta_{prim})$

$\le (1+d^* h^{-\frac12})\left(\log(\delta^{F+pd}) + \log(f^{F+pd})\right) + 200(d^*)^2 h^{\frac12}\log(2d^* h) + 20\eta_{prim}$

(P1) & $80 d_{mod} < d^*_{mod} \le d^*$

(B) $\frac{1}{6}\log(\eta^{*2}) - \frac{1}{6}\log(\eta) \le \frac{1}{6} h^{\frac12}\log \ell \le \frac{1}{3} h^{\frac12}\log(5d^* h) \le h^{\frac12}\log(2d^* h)$

(P3) & (P5) (P5a) $5 < 2^3$

(C) $\frac{1}{6}\log(\eta^{*}) - \frac{1}{6}\log(\eta^{*2}) \overset{?}{\le} B_\kappa$

$\mathbb{R}_{>0}$

(A), (B), (C)

$\frac{1}{6}h = \frac{1}{6}\log(\eta^*) \le (1+d^* h^{-\frac12})\left(\log(\delta^{F+pd}) + \log(f^{F+pd}) + (15d^*)^2 h^{\frac12}\log(2d^* h)\right) + \frac12 C_\kappa$

$\le (1+d^* h^{-\frac12})\left(\log(\delta^{F+pd}) + \log(f^{F+pd}) + \frac16 h \frac25 (60d^*)^2 h^{-\frac12}\log(2d^* h)\right) + \frac12 C_\kappa$

$C_\kappa := 40\eta_{prim} + 2B_\kappa$

$200 < 15^2$

$1 < \frac16 \frac25 4^2$

$\varepsilon_E := (60d^*)^2 h^{-\frac12}\log(2d^* h) \quad (\ge 5d^* h^{-\frac12})$

$\leadsto$ (Eps) $\varepsilon_E \le 4(60d^*)^2 h^{-\frac12}\log(2(d^*)^{\frac14}h^{\frac14}) \le 4(60d^*)^3 h^{-\frac12} h^{\frac14}$

$= 4(60d^*)^3 h^{-\frac14}$

$\forall \varepsilon > 0$ ... If $\varepsilon_E > \min\{1, \varepsilon\}$

$(C) \quad \frac{1}{6}\log(\ldots) \quad 1-\frac{1}{6}\log(\eta^{K}) \le B_K$, $B_K \in \mathbb{R}_{>0}$

$\varepsilon_{\xi} := 40\eta_{\mathrm{prm}} + 2B_K$

$200\,\xi\,15^2$

$\varepsilon_E := (60d^K)^2 h^{-\frac{1}{2}}\log$

$\rightsquigarrow$ (Eps) $\varepsilon_E \le 4(60d^K)^2$

$\forall \varepsilon > 0$,

If $\varepsilon_E > \min\{1, \varepsilon\}$

$\Rightarrow h^{\frac{1}{4}}$: bdd $\;(\leftarrow (\mathrm{Eps}))$

$\Rightarrow$ enlarge $\mathrm{Exc}_{K,d}$ by an $\mathrm{Exc}_{K,d,\varepsilon}$

we may assume $\varepsilon_E \le \min\{1, \varepsilon\}$

$$\Rightarrow \frac{1}{6}h \le \left(1-\frac{2}{5}\varepsilon_E\right)^{-1}\left(1+\frac{1}{5}\varepsilon_E\right)\left(\log(\delta^{F_{\mathrm{tpd}}}) + \log(f^{F_{\mathrm{tpd}}})\right) + \left(1-\frac{2}{5}\varepsilon_E\right)^{-1}\frac{1}{2}C_K$$

$$\le (1+\varepsilon_E)\left(\left(\log(\delta^{F_{\mathrm{tpd}}}) + \log(f^{F_{\mathrm{tpd}}})\right) + C_K\right) + C_K$$

$$\le (1+\varepsilon_E)\left(\log \mathrm{diff}_{\mathrm{pr}}(x_E) + \log\text{-}\mathrm{cond}_{\{0,1,\infty\}}(x_E)\right) + C_K$$

$$\le (1+\varepsilon)\left(\log \mathrm{diff}_{\mathrm{pr}}(x_E) + \log\text{-}\mathrm{cond}_{\{0,1,\infty\}}(x_E)\right)$$

$\leftarrow \varepsilon_E \le \varepsilon$, $\log \mathrm{diff}_{\mathrm{pr}}(x_E) = \log(\delta^{F_{\mathrm{tpd}}})$, $\log(f^{F_{\mathrm{tpd}}}) \le \log\text{-}\mathrm{cond}_{\{0,1,\infty\}}(x_E)$

def of ε_E

$\varepsilon \ge 5d^K h^{-\frac{1}{8}}$

$\frac{1+\frac{1}{5}\varepsilon_E}{1-\frac{2}{5}\varepsilon_E} \le 1+\varepsilon_E$

$1-\frac{2}{5}\varepsilon_E \ge \frac{1}{2}$

$+\ \frac{1}{6}\log(\eta^K) \lesssim \mathrm{ht}_{\omega_{\mathrm{pr}}(0,1,\infty)}$

$\rightarrow$ Prop OK //

$$\tfrac{1}{6} h^{2} \log \ell \leq \tfrac{1}{3} h^{2} \log(5 d^{*} h) \leq h^{\frac{1}{2}} \log(2 d^{*} h)$$

$$\uparrow (p3) \in (p5) \qquad \uparrow (p5a) \qquad \uparrow 5 < 2^{3}$$

(c) $\frac{1}{6}\log(\eta^{\forall}) - \frac{1}{6}\log(\eta^{\forall 2}) \overset{?}{\leq} B_{K}$, $B_K \in \mathbb{R}_{>0}$

$$\tfrac{1}{6} h = \tfrac{1}{6}\log(\eta^{\forall}) \leq (1 + d^{*} h^{-\frac{1}{2}})(\log(\delta^{F_{tpd}}) + \log(f^{F_{tpd}}) + (60 d^{*})^{2} h^{-\frac{1}{2}} \log(2 d^{*} h) \ldots$$

$$\leq (1 + d^{*} h^{-\frac{1}{2}})(\log(\delta^{F_{tpd}}) + \log(f^{F_{tpd}}) + \tfrac{1}{6} \cdot \tfrac{2}{5}(60 d^{*})^{2} h^{-\frac{1}{2}} \log(2 d^{*} h) \ldots$$

$$+ \tfrac{1}{2} C_K$$

$C_K := 40\eta_{prm} + 2B_K$

$200 < 15^2$

$1 < \frac{1}{6} \cdot \frac{2}{5} \cdot 4^2$

$\varepsilon_E := (60 d^{*})^{2} h^{-\frac{1}{2}} \log(2 d^{*} h) \quad (\geq 5 d^{*} h^{-\frac{1}{2}})$

$\leadsto$ (Eps) $\varepsilon_E \leq 4 (60 d^{*})^{2} h^{-\frac{1}{2}} \log(2 (d^{*})^{\frac{1}{4}} h^{\frac{1}{4}}) \leq 4 (60 d^{*})^{2} h^{-\frac{1}{2}} h^{\frac{1}{4}}$

$= 4 (60 d^{*})^{2} h^{-\frac{1}{4}}$

$\forall \varepsilon > 0$, If $\varepsilon_E > \min\{1, \varepsilon\}$

$\Rightarrow h^{\frac{1}{4}}$: bdd ($\leftarrow$ (Eps))

$\Rightarrow$ enlarge $\mathrm{Exc}_{K,d}$ by an $\mathrm{Exc}_{K,d,\varepsilon}$

we may assume $\varepsilon_E \leq \min\{1, \varepsilon\}$

$$\Rightarrow \tfrac{1}{6} h \leq (1 - \tfrac{2}{5}\varepsilon_E)^{-1}(1 + \tfrac{1}{5}\varepsilon_E)(\log(\delta^{F_{tpd}}) + \log(f^{F_{tpd}}))$$

$$+ (1 - \tfrac{2}{5}\varepsilon_E)^{-1} \tfrac{1}{2} C_K$$

$$\leq (1 + \varepsilon_E)(\log(\delta^{F_{tpd}}) + \log(f^{F_{tpd}})) + C_K$$

$$\leq (1 + \varepsilon_E)(\log\text{-diff}_{pr}(x_E) + \log\text{-cond}_{\{0,1,\infty\}}(x_E)) + C_K$$

$$\leq (1 + \varepsilon)(\log\text{-diff}_{pr}(x_E) + \log\text{-cond}_{\{0,1,\infty\}}(x_E))$$

$\varepsilon_E \leq \varepsilon$, $\log\text{-diff}_{pr}(x_E) = \log(\delta^{F_{tpd}})$, $\log(f^{F_{tpd}}) \leq \log\text{-cond}_{\{0,1,\infty\}}(x_E)$

def of ε_E

$\varepsilon \geq 5 d^{*} h^{-\frac{1}{2}}$

$\frac{1 + \frac{1}{5}\varepsilon_E}{1 - \frac{2}{5}\varepsilon_E} \leq 1 + \varepsilon_E$

$1 - \frac{2}{5}\varepsilon_E \geq \frac{1}{2}$

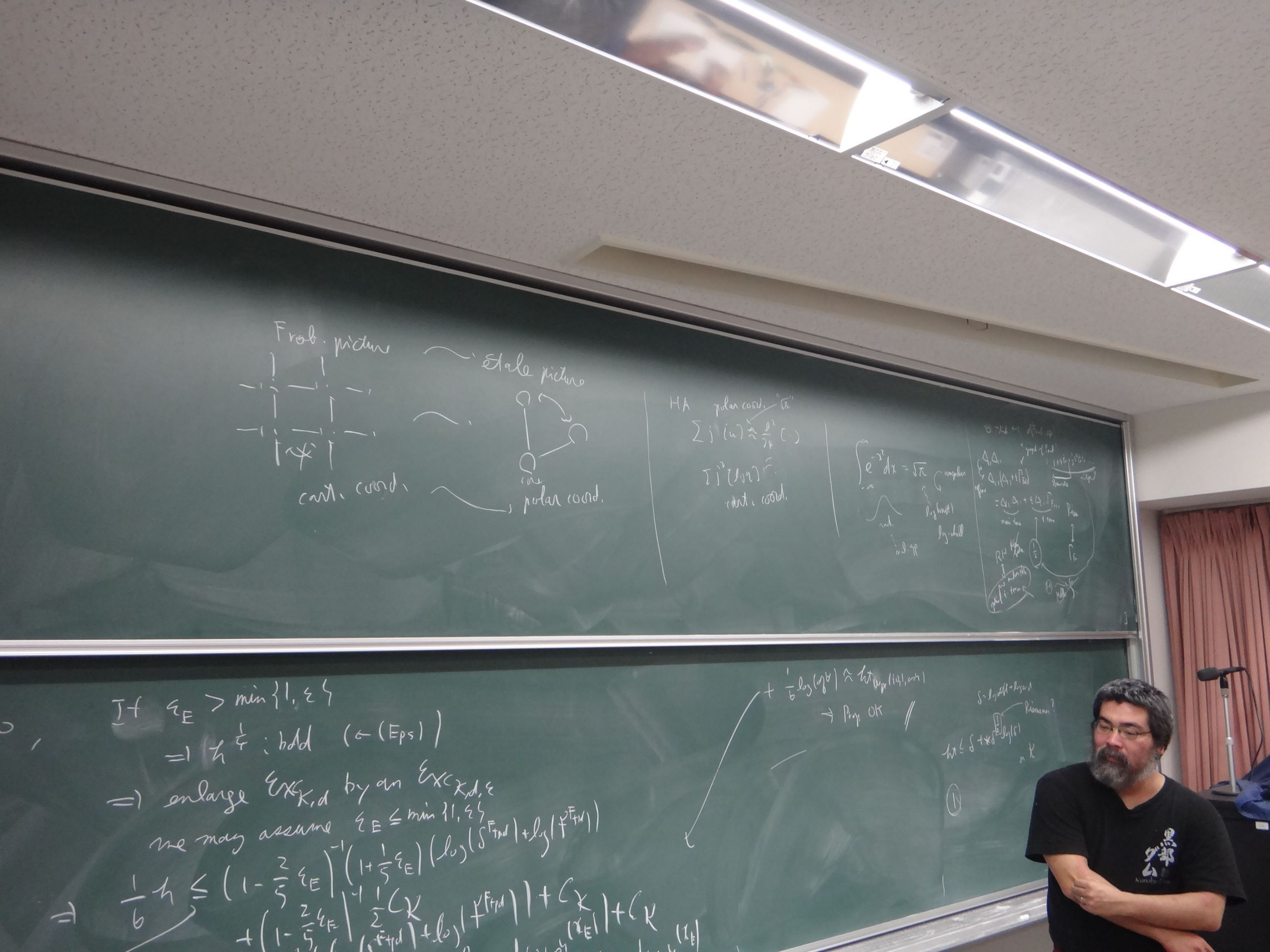
$+ \frac{1}{6}\log(\eta^{\forall}) \approx \mathrm{ht}_{\omega_{pr}(\{0,1,\infty\})}$

$\Rightarrow$ Prop. OK //

$\delta = \log\text{-diff} + \log\text{-cond}$

$\mathrm{ht} \leq \delta + * \delta^{\boxed{\frac{1}{2}}} \log(\delta)$

Riemann?



Frob. picture
étale picture
cart. coord.
polar coord.
HA
polar coord.
cart. coord.
$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$
If $\varepsilon_E > \min\{1, \varepsilon\}$
$\Rightarrow h^{\frac{1}{4}}$: bdd ($\leftarrow$ (Eps))
$\Rightarrow$ enlarge $Exc_{K,d}$ by an $Exc_{K,d,\varepsilon}$
we may assume $\varepsilon_E \leq \min\{1, \varepsilon\}$
→ Props OK
Riemann?


平成27年3月 9日(月)～13日(金): 420
平成27年3月16日(月)～20日(金): 110
RIMS共同研究
宇宙際タイヒミューラー理論の
検証と更なる発展
京都大学・数理解析研究所
望月 新一 氏

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RIMS共同研究

宇宙際タイヒミューラー理論の検証と更なる発展

京都大学・数理解析研究所
望月　新一　氏

共同利用掛

RIMS 共同研究（特別計画）「宇宙際タイヒミューラー理論の検証と更なる発展」

京都大学数理解析研究所の共同利用・共同研究事業の一つとして、下記の RIMS 共同研究を開催します。

提案者：望月新一（京大数理研）
代表者：望月新一（京大数理研）
世話人：田口雄一郎（九大数理）

記

期間：平成２７年３月９日（月）〜２０日（金）
場所：京都大学数理解析研究所４２０号室（　９〜１３日）
　　　　　　　　　　　　　　１１０号室（１６〜２０日）

３月９日（月）

09:30 〜 10:50 星裕一郎（京大数理研）
「数体の単遠アーベル的復元」

11:00 〜 12:30 山下剛（京大数理研）
「宇宙際 Teichmüller 理論とその Diophantus 的帰結」

14:00 〜 18:00 山下剛（京大数理研）
「宇宙際 Teichmüller 理論とその Diophantus 的帰結」

３月１０日（火）〜１３日（金）
　　１６日（月）〜２０日（金）

09:30 〜 12:30 山下剛（京大数理研）
「宇宙際 Teichmüller 理論とその Diophantus 的帰結」

14:00 〜 18:00 山下剛（京大数理研）
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注：３月９日（月）または１０日（火）に開催予定の Minhyong Kim 教授（Oxford 大学）の講演（１時間）のため、上記 RIMS 共同研究の講演を一時中断する方向で考えております。

講演要旨集

星裕一郎（京大数理研）「数体の単遠アーベル的復元」

Neukirch・内田の定理によって、数体（つまり、有理数体の有限次拡大）の同型類はその絶対 Galois 群の位相群としての同型類によって完全に決定されることがわかります。一方、Neukirch・内田の定理（あるいはその証明）は、絶対 Galois 群から元の数体そのものを記述する「純群論的な手続き」を与えません。この講演では、数体の絶対 Galois に対するそのような手続きについてお話したいと思います。

山下剛（京大数理研）「宇宙際 Teichmüller 理論とその Diophantus 的帰結」

2012年8月、望月新一氏（京大数理研）は宇宙際 Teichmüller 理論の連続論文（I〜IV）を発表した。これは、きわめて大雑把に述べると、スキーム論の外に出て数体の「数論的正則構造」を「変形」し、絶対遠 Abel 幾何的復元アルゴリズムを使うことで一方の「数論的正則構造」から他方の「数論的正則構造」を軽微な不定性を許して復元し、その帰結として Diophantus 不等式を導くというものである。不定性が軽微なもので抑えられることを示すところ（や「変形」の構成など）において、理論中に出てくる数学的部品たちの性質が絶妙にピタリとあてはまっている。

同氏は、その理論の準備の段階の論文を含め、「単遠 Abel 幾何と双遠 Abel 幾何」「数論的正則性と単解析性」「エタール的対象と Frobenius 的対象」「多輻性と単輻性と核性」「足し算と掛け算を分離する数論的な上半平面」「数論的な解析接続」「Galois 評価原理」などの（重要かつ整理された視点を提供する）独創的な数学的概念・視点を導入し、全く新しい地平を切り開いた。これは Diophantus 不等式への応用抜きにしてもそれ自身重要かつ有用な概念・視点である（また、これら以外にも多くの興味深い対応関係や対比がある）。

本連続講演は、理論全体の概観の後、理論の思想的源流（Hodge-Arakelov 理論や p 進 Hodge 理論など）について簡単に触れ（同氏の導入した概念や理論は単に新奇であるのではなく、よく理解すれば Gauss 積分やテータ関数の Jacobi の等式などの古典的な理論と思想的に深く結びついている）、準備の論文の解説（Belyi カスプ化や楕円テータ環境の３つの剛性など）をして、本体の論文（キーワードだけを並べると、種々の Hodge 劇場、種々のテータ・リンクと Hodge-Arakelov 理論的評価、対数的殻と対数的リンク、対数的 Kummer 対応、多輻的復元アルゴリズム、対数的体積計算など）に進む予定である。

なお、宇宙際 Teichmüller 理論は現在詳細の検証中にある。論文発表以降、検証作業は秘匿性のない形においても専門家たちにより良好に進められており肯定的に受け取られているが、何重にもおよぶ検証作業はまだ完全には終わっておらず、この連続講演は検証作業が完了したことを意味するわけではない。